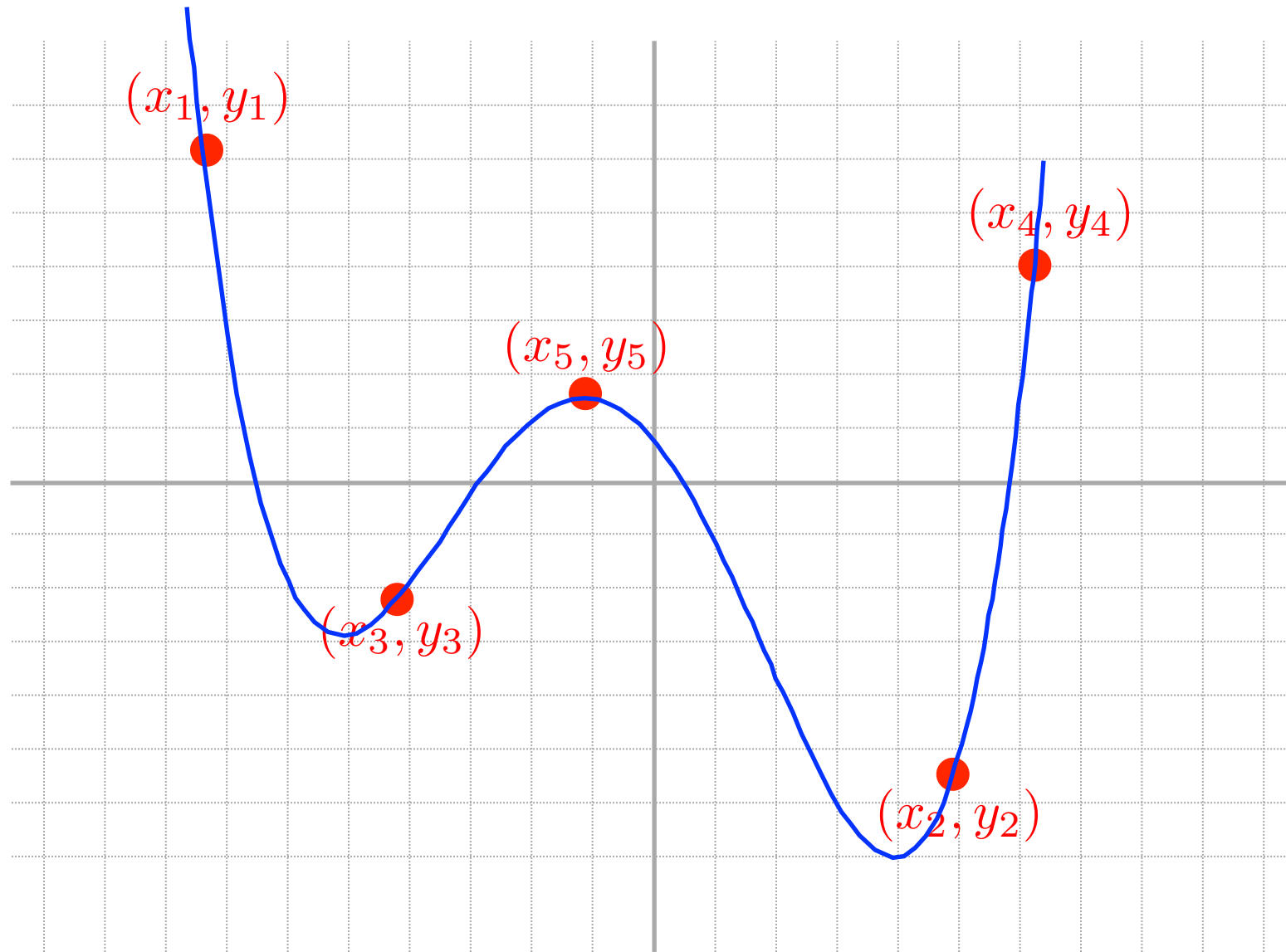


Application: l'interpolation

Trouver un polynôme de degré ≤ 4 qui passe par les points ci-dessous



On cherche une fonction $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ dont le graphe passe par les points donnés

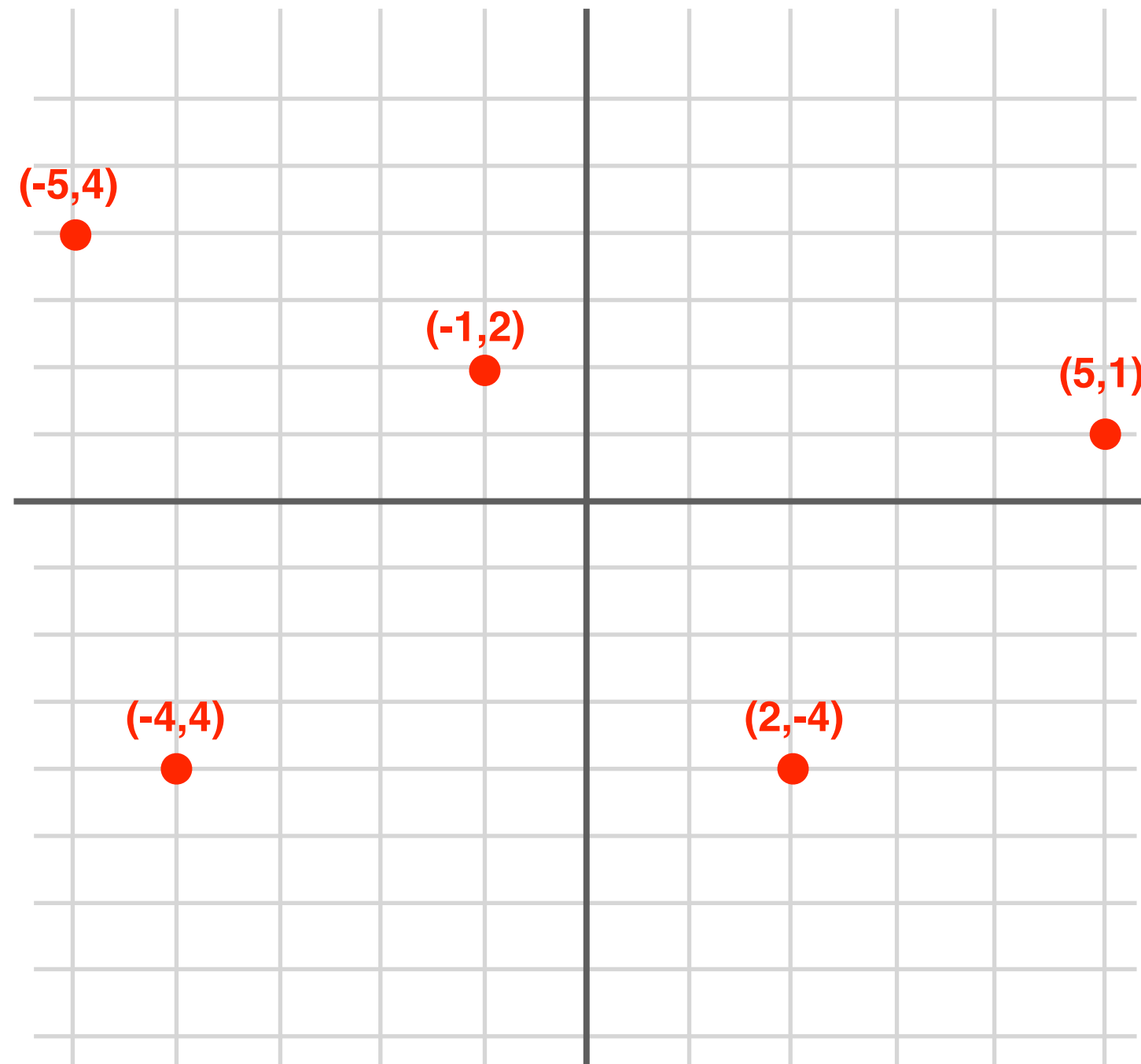
Système linéaire

$$\begin{aligned}
 ax_1^4 + bx_1^3 + cx_1^2 + dx_1 + e &= y_1 \\
 ax_2^4 + bx_2^3 + cx_2^2 + dx_2 + e &= y_2 \\
 ax_3^4 + bx_3^3 + cx_3^2 + dx_3 + e &= y_3 \\
 ax_4^4 + bx_4^3 + cx_4^2 + dx_4 + e &= y_4 \\
 ax_5^4 + bx_5^3 + cx_5^2 + dx_5 + e &= y_5
 \end{aligned}$$

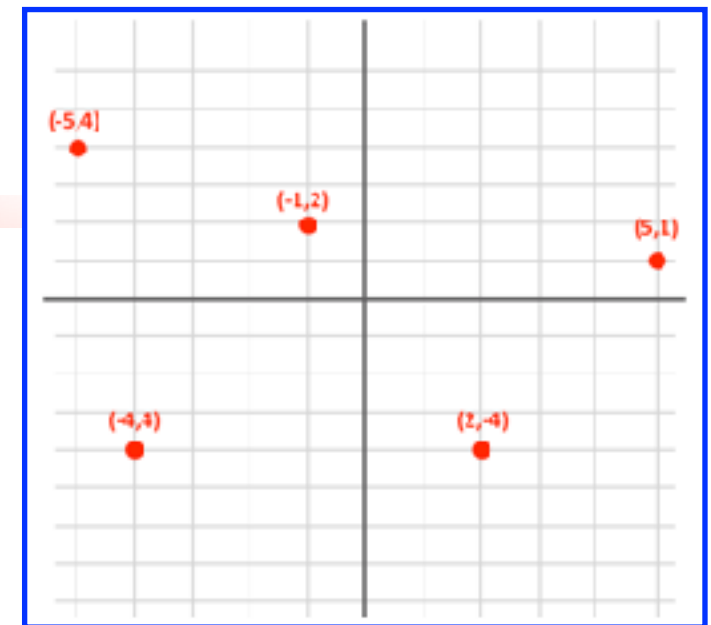
$$\begin{array}{ccc}
 \text{connu} & & \text{inconnu} \quad \text{connu} \\
 \left(\begin{array}{ccccc} x_1^4 & x_1^3 & x_1^2 & x_1 & 1 \\ x_2^4 & x_2^3 & x_2^2 & x_2 & 1 \\ x_3^4 & x_3^3 & x_3^2 & x_3 & 1 \\ x_4^4 & x_4^3 & x_4^2 & x_4 & 1 \\ x_5^4 & x_5^3 & x_5^2 & x_5 & 1 \end{array} \right) & \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} = & \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix}
 \end{array}$$

Y a-t-il une solution?

Exemple



Exemple



$$x_1 = -5, x_2 = -4, x_3 = -1, x_4 = 2, x_5 = 5$$

$$y_1 = 4, y_2 = -4, y_3 = 2, y_4 = -4, y_5 = 1$$

système d'équations linéaires

$$ax_1^4 + bx_1^3 + cx_1^2 + dx_1 + e = y_1$$

$$ax_2^4 + bx_2^3 + cx_2^2 + dx_2 + e = y_2$$

$$ax_3^4 + bx_3^3 + cx_3^2 + dx_3 + e = y_3$$

$$ax_4^4 + bx_4^3 + cx_4^2 + dx_4 + e = y_4$$

$$ax_5^4 + bx_5^3 + cx_5^2 + dx_5 + e = y_5$$

$\Rightarrow$

$$\begin{pmatrix} \textcolor{red}{a} & \textcolor{green}{b} & \textcolor{blue}{c} & \textcolor{orange}{d} & e & | & \end{pmatrix} \begin{pmatrix} 625 & -125 & 25 & -5 & 1 & | & 4 \\ 256 & -64 & 16 & -4 & 1 & | & -4 \\ 1 & -1 & 1 & -1 & 1 & | & 2 \\ 16 & 8 & 4 & 2 & 1 & | & -4 \\ 625 & 125 & 25 & 5 & 1 & | & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & | & \boxed{337/5670} \\ 0 & 1 & 0 & 0 & 0 & | & \boxed{131/5670} \\ 0 & 0 & 1 & 0 & 0 & | & \boxed{-67/45} \\ 0 & 0 & 0 & 1 & 0 & | & \boxed{-2488/2835} \\ 0 & 0 & 0 & 0 & 1 & | & \boxed{1460/567} \end{pmatrix}$$

Polynôme:

$$\textcolor{red}{\boxed{337/5670}}x^4 + \textcolor{green}{\boxed{131/5670}}x^3 + \textcolor{blue}{\boxed{-67/45}}x^2 + \textcolor{orange}{\boxed{-2488/2835}}x + \textcolor{blue}{\boxed{1460/567}}$$

Exemple

$$\frac{337}{5670}x^4 + \frac{131}{5670}x^3 - \frac{67}{45}x^2 - \frac{2488}{2835}x + \frac{1460}{567}$$

